# Miniaturization of Microstrip Patch Antenna Using Fractal Geometry

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This paper demonstrates the miniaturization of microstrip patch antenna through the induction of fractal geometry.

This paper demonstrates the miniaturization of microstrip patch antenna through the induction of fractal geometry. By using the space filling properties of fractals, a unique patch antenna operating at 2.20 GHz is achieved as compared to conventional antenna which is operating at 2.46 GHz frequency, this is due to the increase of electrical length which is inversely proportional to operating frequency. The design starts with an insert-fed patch antenna which is subjected to fractal iteration using a new version of Sierpinski fractal up to the 3rd iteration. Experimental results show size reduction of 31%, 32% and 33% for 1st, 2nd and 3rd iterations respectively. Not only size is reduced but also the antenna maintains performance characteristics such as reflection loss, impedance matching, antenna gain. Additionally, the propose design achieves an increase bandwidth of over 3%, which illustrates the performance get better while maintaining the frequency. The results are evidence that proposed design provides superior size reduction and improved the bandwidth to previous design.

Keywords - Fractal antenna, Inset-fed, Microstrip Patch Antenna, Miniaturization, Sierpinski Carpet

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## 1. INTRODUCTION

The Microstrip patch antennas (MPAs) have become a base for modern wireless communication systems due to many advantages, such as low profile, lightweight, ease of integration with both planar and non-planar surfaces, and cost-effective manufacturing using printed circuit technology [1]. With the rapid expansion of wireless applications, there is an increasing demand for smaller antennas. Traditional MPAs typically have dimensions around half the wavelength, prompting the development of various miniaturization techniques to reduce these sizes. Key approaches include the use of high-permittivity substrates [2], magnetic substrates [3], increasing electrical length, reactive loadings, short-circuiting, and incorporating superstrates [4]. In addition to these methods, fractal geometry has emerged as a novel approach to antenna miniaturization [5].

Fractals, characterized by their irregular and self-replicating structures, were first introduced by the French mathematician Benoît Mandelbrot. Unlike Euclidean geometry, which represents natural forms like mountains and clouds as simple cones and ellipsoids, fractal geometry provides a more accurate representation of the complex shapes found in nature. Fractals are composed of repeated iterations of a basic shape, with scaled-down versions forming the building blocks of the overall structure. This concept has found its uses in various fields, including image compression, weather prediction, integrated circuits, filter design, and particularly in the emerging field of fractal electrodynamics, where fractal geometry is applied to electromagnetic theory to explore new possibilities in radiation, propagation, and scattering phenomena [6].

In antenna design, the self-similarity and space-filling properties of fractals are particularly important.

Self-similarity involves smaller copies of the entire structure within the same design, while self-affinity describes similar features with different scale factors in different directions. Space-filling properties allow fractal designs to occupy less physical space while maintaining electrical length. These characteristics have been utilized in designing multi-frequency antennas, such as the Sierpinski Gasket, and in developing compact antennas, like the Sierpinski Carpet [7].

Various fractal shapes, including the Sierpinski Carpet, Sierpinski Gasket, Minkowski Loop, and Koch Island, have been explored to achieve size reduction in patch antennas. For instance, a microstrip antenna with Koch-shaped fractal defects has been proposed to achieve a smaller size, while another design incorporates Sierpinski Carpets and Koch curves for further miniaturization [8]. The Koch island fractal boundary and fractal butterfly patch antennas are other examples of successful fractal-based miniaturized antenna designs [9].

This paper explores the application of fractal geometry to optimize the shape of conventional patch antennas for size reduction. Specifically, it investigates the use of the Sierpinski Carpet, a simple space-filling fractal, in different iteration orders on an inset-fed microstrip patch antenna, which is known for its effective impedance matching [10]. The remainder of the paper is structured as follows: Section II discusses the fundamental concepts behind the proposed antenna design, Section III presents the simulation results of the modified Sierpinski Carpet Microstrip Patch Antenna (SCMPA), and Section IV concludes the paper [11].

## 2. LITERATURE REVIEW

## 2.1 Introduction

Microstrip patch antennas, renowned for their compact size, ease of fabrication, and versatility, have become indispensable in wireless communication systems. However, their relatively large size at lower frequencies poses a significant challenge. To address this, researchers have turned to fractal geometries, which offer a unique approach to miniaturization without compromising performance. Fractal antennas, with their self-similar patterns and extended current paths, effectively reduce resonant frequency while maintaining a compact footprint. This makes them ideal for applications where space is limited [12].

A variety of fractal geometries, including the Koch curve, Sierpinski gasket, and Hilbert curve, have been explored for antenna miniaturization. Each offers distinct advantages in terms of size reduction, multi-band capabilities, and radiation characteristics. For instance, Sierpinski gasket antennas have demonstrated dual-band operation, while Koch and Hilbert curve antennas have achieved substantial size reduction and improved bandwidth [12], [13].

The degree of miniaturization achievable with fractal antennas varies depending on the specific fractal design and the number of iterations employed. Studies have shown miniaturization ratios exceeding 70%, often accompanied by enhanced impedance matching and radiation efficiency. Although fractal antennas may exhibit slightly reduced radiation efficiency compared to larger conventional antennas, their ability to significantly reduce size makes them a compelling option for modern wireless systems where space is a premium [14].

This literature review will delve into the various fractal geometries used for microstrip patch antenna miniaturization, analyzing their advantages, limitations, and performance metrics. The goal is to provide a comprehensive overview of the current state of fractal-based antenna design, highlighting the most effective techniques for optimizing size, bandwidth, and radiation efficiency.

## 2.2 History of Fractal Geometry

Fractals are frequently seen in nature, including on mountains, coasts, and in plants. They have been explored theoretically. Fractal patterns may be seen in traditional African architecture in the manner that smaller buildings mimic bigger ones. Fractals are mathematical structures that have been used to simulate intricate natural formations such as coastlines, snowflakes, and trees. The concept originated from the study of patterns seen in nature. Nature has been refining these fractal patterns over millions of years to maximize their efficiency; this idea has been used to a variety of structures, including antennas.

## 2.3 Fractal Geometry and Fractal Antennas



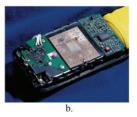


Figure 2.2: Antenna in GPS handset and PIFA Antenna

## 2.4 Types of Fractal Geometry

## 2.4.1 Koch Snowflake

A well-known fractal is the Koch snowflake, which is made by continually adding smaller triangles to the sides of a big ger triangle.Helge von Koch, a Swedish mathematician, firs t presented it in 1904.An equilateral triangle serves as the s tarting point for construction, and smaller triangles are plac ed to each side's center.An indefinite repetition of this proce ss results in a structure that is self-similar and has the same appearance at all scales.

| Number of sides (6) | 1 | 1/3 | 1/8 | 1/92 | 1/94 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97 | 1/97

## 2.4.2 Koch Curve

Helge von Koch also introduced the Koch curve, which is another kind of fractal. It enhances the radiation resistance, bandwidth, and resonant frequency of the antenna. The construction begins with a straight line that is split into three segments. Two of the segments create an equilateral triangle in the middle of the original line. The curve is created by continually repeating this technique for each section of a straight line. Because of their fractal architecture, fractal antennas—such curves—offer multiband as Koch capabilities and exhibit good performance across a wide frequency range. The iterations of the Koch curve are shown in Figure 2.3.

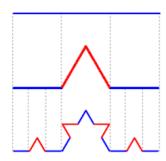


Figure 2.3: Koch Curve fractal iterations

## 2.4.3 Sierpinski Gasket

The Polish mathematician Sierpinski is the name of the Sierpinski triangle, which is often referred to as the Sierpinski gasket. A tiny inverted triangle is taken out of the middle of the first giant triangle in the pattern. If the procedure is carried out indefinitely on the remaining triangles, the structure will create a perfect fractal. Its self-similar structure's ability to function consistently at many sizes makes it very helpful in antenna designs. The Sierpinski gasket iterations are seen in Figure 2.4.

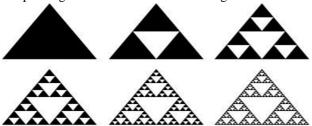


Figure 2.4: Koch Curve fractal iterations [15].

#### 3. ANALYSIS AND DESIGN

## 3.1 REFERENCE ANTENNA: RECTANGULAR PATCH DESIGN & ITS RESULTS

As shown in Figure 1 (a), a rectangular microstrip patch antenna with an inset feed was chosen as the reference antenna. Table 1 contains an inventory of the associated dimensions. An FR-4 substrate ( $\varepsilon_r = 4.3$ ,  $\tan \delta = 0.019$ ) with a thickness of 1.6 mm is used in the antenna. To improve the impedance matching, an inset feed is utilized. The patch and the ground are both composed of copper, which has a 0.035 mm thickness. A waveguide port is used to feed the antenna.

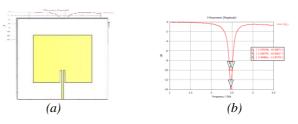


Figure 1: (a) reference antenna and (b) reference antenna

The S-parameter displayed in Figure 1 (b) shows that the reference antenna resonates at  $2.47~\mathrm{GHz}$  with a return loss of  $-13.86~\mathrm{dB}$ , operating within the frequency range of  $2.44~\mathrm{to}$   $2.5~\mathrm{GHz}$ 

Table 1: Parameters of the reference antenna Fractal geometry

S.No	Antenna Parameters	Dimensions	
1	Length of substrate	47.08mm	
2	Width of substrate	54.6mm	
3	Width of ground	54.6mm	
4	Length of ground	47.08mm	
5	Thickness of substrate	1.6mm	
6	Thickness of ground	0.035mm	
7	Length of patch	27.88mm	
8	Width of patch	35.4mm	
9	Thickness of patch	0.035mm	
10	Length of feed line	16.60mm	
11	Width of feed line	01mm	
12	Thickness of feed line	0.035mm	

## 3.1.1 VSWR

The Voltage Standing Wave Ratio (VSWR) value should be between 1 and 2 in order to maximize the amount of energy transferred from the feed line to the antenna. The power reflected back in the transmission line, which represents the matching or mismatching of the antenna, is indicated by the VSWR. The rectangular microstrip patch antenna's VSWR is

depicted in Figure 2 (a), where it is 1.51 at the resonance frequency.

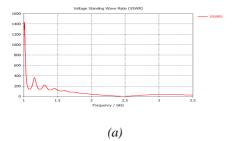


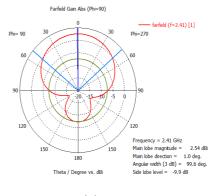
Figure 2: (a) reference antenna VSWR

## 3.1.2 RADIATION PATTERN

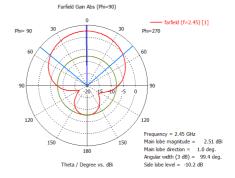
Directivity of an antenna defined as "the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions."

The radiation patterns at the lower cut-off frequency, resonance frequency, and upper cut-off frequency are

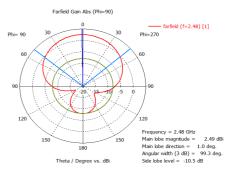
illustrated in Figure 3 (a), (b), and(c),respectively. The results as shown in table 2.



*(a)* 



(b)



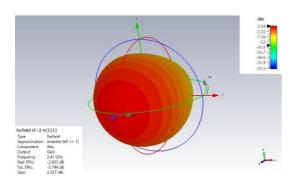
(c)

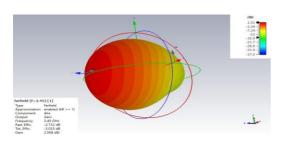
Figure 3: (a) radiation pattern at lower cut-off frequency, (b) radiation pattern resonance frequency and (c) radiation pattern upper cut-off frequency

## **3.1.3 GAIN**

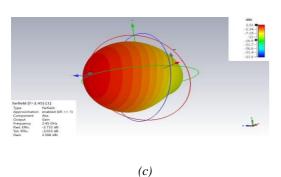
For the performance of an antenna another useful describing measure is Gain. According to IEEE Standards, "gain does not include losses arising from impedance mismatches and polarization mismatches.

The gain at at the lower cut-off frequency, resonance frequency, and upper cut-off frequency are illustrated in Figure 4 (a), (b), and(c),respectively. The results as shown in table2.





(a)



(b)

Figure 4: (a) gain at the lower cut-off frequency, (b) gain at resonance frequency, (c) gain at upper cut-off frequency

## 3.2 ANTENNA WITH FRACTAL GEOMETRY

The zeroth iteration of the design is represented by a rectangular patch at the beginning. Several iterations of the Sierpinski Carpet geometry were used to create the suggested

antenna. In the beginning, the rectangular patch is built in one go, as Figure 5 (a), (b) and (c) illustrates. After then, this patch goes through several versions. The patch is split into  $\frac{1}{3}$ , ninths  $\frac{1}{9}$ , and one twenty-seventh  $\frac{1}{27}$  in the 1st, 2nd, and 3rd iterations, respectively. As Figure 5 (a), (b) and (c) illustrates, this recursive process keeps producing higher-order iterations.

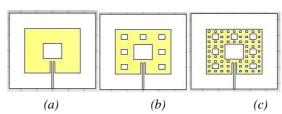


Figure 5: (a)  $1^{st}$  iteration, (b)  $2^{nd}$  iteration (c)  $3^{rd}$  iteration

## 3.2.1 ITERATIONS RESULTS

## 3.2.1.1 RETURN LOSS

The return loss of the rectangle patch MPA of the  $1^{st}$ ,  $2^{nd}$  and  $3^{rd}$  iterations are shown in Figure 6 (a), (b) and (c), respectively. The return loss results are shown in table 1.

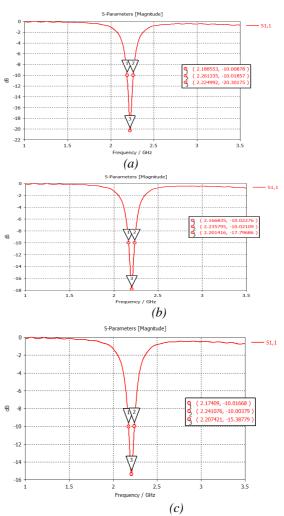


Figure 6: (a)  $1^{st}$  iteration S11, (b)  $2^{nd}$  iteration, and (c)  $3^{rd}$  iteration

## 3.2.1.2 VSWR

Figure 7 (a), (b) and (c) shows the VSWR of rectangular patch MPA with 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> iterations are 1.21, 1.3, and 1.41 at 2.2 GHz, respectively.

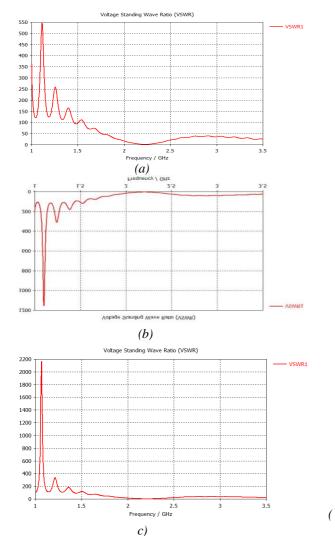
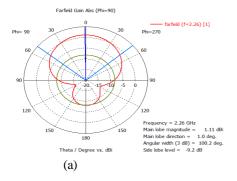


Figure 7: (a) 1<sup>st</sup> Iteration VSWR, (b) 2<sup>nd</sup> iteration VSWR (c) 3<sup>rd</sup> Iteration VSWR

## 3.2.1.3 RADIATION PATTERNS

The radiation patterns of the rectangle patch MPA the 1st , 2nd and 3rd iterations are shown in Figure 8 (a), (b) and (c) , respectively. The results are shown in table 1.



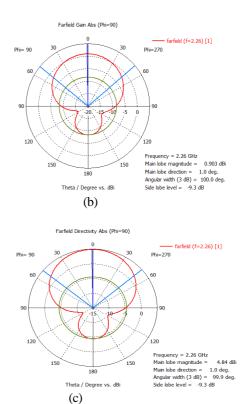
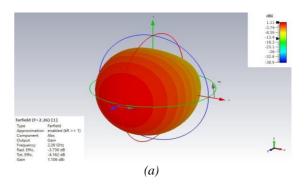


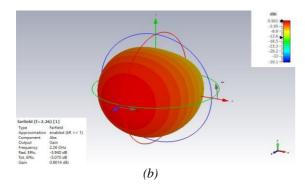
Figure 8: (a) 1st Iteration Radiation Pattern, (b) 2nd iteration Radiation Pattern, 3rd Iteration Radiation Pattern

### 3.2.1.4 GAIN

An antenna's three-dimensional radiation pattern is seen in the image. This pattern illustrates how the antenna sends energy in various directions and displays the spatial distribution of radiated power. The pattern shows the efficiency and gain at particular frequencies. The colors show how powerful the radiation is; blue indicates weaker radiation and red indicates greater radiation. The resonant frequency, gain, and overall efficiency of the antenna are included in the data that goes with the pattern, demonstrating how effectively it operates within the specified frequency range. Understanding the antenna's performance in practical applications—such as its capacity to direct energy in a specific direction its overall or transmission efficiency—requires a grasp of both the data and the visual depiction.

The gains of the rectangle patch MPA the 1st, 2nd and 3rd iterations are shown in Figure 9 (a), (b) and (c), respectively. The results are shown in table 1.





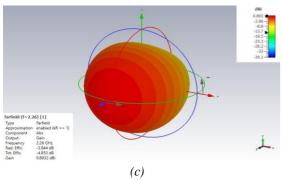


Figure 9: (a)  $1^{st}$  iteration Gain, (b)  $2^{nd}$  iteration Gain, and  $3^{rd}$  iteration Gain

## 3.2.1.5 COMPARISON AND ANALYSIS OF ALL RESULTS

The suggested antenna's iterative design method shows a noticeable improvement in terms of resonant frequency tuning and size reduction. As can be seen in Figure 9, as more iterations are carried out, the physical size of the antenna decreases while it continues to operate efficiently within the intended frequency range. Significantly, a movement to the left in the resonating frequencies indicates a decrease in frequency. Because of the inverse relationship between frequency and antenna size, each iteration of the antenna essentially increases the working wavelength. This is consistent with the rise in the electrical length of the antenna. The antenna performs best at the desired frequency of 2.20 GHz, which was selected to satisfy particular communication standards. The antenna's physical size would, however, significantly grow with further working frequency decrease, underscoring the trade-off between size and frequency performance. The CST Microwave Studio simulation results confirm that the suggested design is efficient in obtaining optimal resonance and compactness.

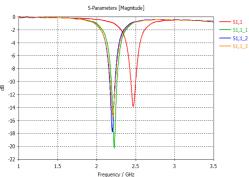


Fig 6: Overall Return Loss Results

1				
Antenna	Iter 0	Iter 1	Iter 2	Iter 3
Parameters				
S11 (dB)	-13.85	-20.30	-17.79	-15.38
VSWR	1.51	1.21	1.3	1.41
Gain (dB)	2.49	1.11	0.91	0.893
D	5.3	4.84	4.84	4.84
$f_{op}$	2.47	2.23	2.21	2.20
BW%)	2.49	3.27	3.13	3.03
<i>IBW</i> (%)	62 мнz	73 мнz	69 мнz	67 MHz

Table 2: Results of different iterations

Antenna	Iter 0	Iter 1	Iter 2	Iter 3
Area	986.95	680.54	675.91	670.5
Mini.	XX	31%	32%	33%

Table 3: Dimensions of Antenna

## 4. CONCLUSION

In this research we achieved the design of miniaturized novel patch antenna operating at 2.20 GHz. The patch of antenna is split to one twenty seven (1/27) in the 3rd iteration, achieving 33% of size reduction. The performance of antenna is maintained along with the desired result.

## 5. REFERENCES

- [1]. Ullah, Habib, et al. "Angularly Stable Band Stop FSS Loaded MIMO Antenna with Enhanced Gain and Low Mutual Coupling." Int. J. Advanced Networking and Applications 15.03 (2023): 5924-5930.
- [2]. T. K. Lo and Y. Hwang," Microstrip Antennas of Very High Permittivity for Personal Communications," 1997 Asia Pacific Microwave Conference, pp. 253–256.
- [3]. H. Mosallaei and K. Sarabandi," Design and Modeling of Patch AntennaPrintedon Magneto -Dielectric Embedded-Circuit Metasubstrate," IEEE Transactions On Antennas And Propagation, Vol. 55, No. 1, January 2007.
- [4]. N. G. Alexopoulos and D. R. Jackson, "Fundamental Superstrate (Cover) Effects On Printed Circuit Antennas," IEEE Transactions On Antennas And Propagation, Vol. Ap-32, No. 8, August 1984.
- [5]. J. P. Gianvittorio and Y. Rahmat-Samii, "Fractal Antennas: A Novel Antenna Miniaturization Technique, and Applications," IEEE Antenna's and Propagation Magazine, Vol. 44, No. 1, February 2002.
- [6]. J. Anguera, C. Puente, C. Borja, R. Montero, and J. Soler," Small And High-Directivity Bow-Tie Patch

- Antenna Based On The Sierpinski Fractal," Microwave And Optical Technology Letters ,Vol. 31, No. 3, November 5 2001.
- [7]. Anguera, Jaume, et al. "Broad-band dual-frequency microstrip patch antenna with modified Sierpinski fractal geometry." IEEE Transactions on Antennas and Propagation 52.1 (2004): 66-73.
- [8]. Kordzadeh, Atefeh, and Farrokh Hojjat-Kashani. "A new reduced size microstrip patch antenna with fractal shaped defects." Progress In Electromagnetics Research B 11 (2009): 29-37.
- [9]. Mohammed, S. Sheik, K. Ramasamy, and T. Shanmuganantham. "A sierpinski fractal based microstrip patch antenna for wireless power transmission system." International Journal of Computer Applications 1.13 (2010): 0975-8887.
- [10]. Wong, Hang, et al. "Small antennas in wireless communications." Proceedings of the IEEE 100.7 (2012): 2109-2121.
- [11]. Balanis, C. A. Antenna Theory: Analysis and Design, 3rd ed. Wiley, 2005.
- [12]. J. P. Gianvittorio and Y. Rahmat-Samii, "Fractal Antennas: A Novel Antenna Miniaturization Technique, and Applications," IEEE Antenna's and Propagation Magazine, Vol. 44, No. 1, February 2002.
- [13]. J. Anguera, E. Martínez, C. Puente, C. Borja, and J. Soler, "Broad-Band Dual-Frequency Microstrip Patch Antenna With Modified Sierpinski Fractal Geometry," IEEE Transactions On Antennas And Propagation, Vol. 52, No. 1, January 2004.
- [14]. S. Sheik Mohammed, K. Ramasamy, and T. Shanmuganantham, "A Sierpinski Fractal Based Microstrip Patch Antenna for Wireless Power Transmission System," ©2010 International Journal Of Computer Applications (0975 8887), Volume 1 No. 13.
- [15]. Anguera, Jaume, et al. "Broad-band dual-frequency microstrip patch antenna with modified Sierpinski fractal geometry." IEEE Transactions on Antennas and Propagation 52.1 (2004): 66-73.